Fast Bipedal Walk Using Large Strides by Modulating Hip Posture and Toe-heel Motion

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Abstract—Typically most humanoid robots walk with relatively small strides even on the level ground, and consequently their walking speed is fairly slow compared to humans. One reason is that the constraint of the constant COM (Center of Mass) height, which is for decoupling the frontal and lateral motion, produces the characteristic bent knee feature of walking robots and requires higher motor torques. The other reason is the feet trajectory limitation which demands the feet remain parallel to the ground. The parallel foot placement feature, using no toe-off and heel-strike motions, means that robots have to lower their hip height to perform large strides. This paper studies a trajectory generation method that enables fast bipedal walking with large strides. We address this issue by formulating a hip pattern generator and a feet trajectory generator with toe-off and heel-strike motions, based on the preview control. This scheme is applied to a dynamic simulation of the child humanoid "iCub", which demonstrates successful gaits in large strides at the average speed from 1.08 km/h to 2.52 km/h.

I. INTRODUCTION

Humanoids are able to perform a variety of stable walking tasks [1], [2], [3], [4], [5]. Nevertheless, most of them walk with relatively small strides and slow speeds compared to humans. However, to produce fast robot walking with a given gait cycle, robots need to take larger strides.

To date, relatively little work has been done on fast walking by increasing stride length, although there are two significant benefits. Firstly, there is an obvious faster walking speed given the same gait frequency. Secondly, there is also the capacity to avoid or step over wider obstacles and potentially deal with rough or broken terrain. Stasse, et al. [6] showed that the increased strides could help significantly by stepping over obstacles but the speed was relatively slow in their experiments.

However, there are many factors that limit the fast walking and two of them are investigated in this study. One is the constraint of a constant COM height, and other is the feet trajectory that keeps feet level to the ground. From the control point of view, imposing a constant COM height (or hip height) greatly simplifies control challenges. By setting a constant COM height [7], [8] or a constant hip height [9], the ZMP (zero moment point) [10] equation can be decoupled in the sagittal and frontal plane respectively. Hence, the ZMP equations can be represented by the state space equations with well approximated dynamics [7]. The above advantage makes the constant COM/hip height constraint the most widely applied technique in humanoids walking. However, to perform large strides, the hip height needs to be particularly lower to avoid singularities near the full extension of the knees. Besides, although the flat-feet walking, which restricts the feet to be horizontal, provides larger stability margins, it makes the robot difficult to take large strides. Without a toe-off and heel-strike motion, the robot needs to lower its hip height even more to avoid the knee singularities. If the above mentioned constraints remain, it inevitably results in the characteristic highly bent knees found in most humanoids, resulting in a graceless walking manner, high motor torques, and therefore inefficient walking.

Ogura, et al. [11] successfully achieved a more human-like walking with stretched knees, toe-off and heel-contact for the WABIAN-2R robot. It uses the predetermined knee joint trajectories consisting of two sinusoidal motions to generate the stretched knees feature, and a pattern generator to create the feet trajectories. The inverse kinematics is solved by using an extra 2-DoF waist. Its hip height is variable as a result of prescribed knees but not purposely designed. Therefore, hip trajectory is not sinusoidal as in humans. Its walking speed is up to 1.875 km/h, with a stride length of 1 m and a gait cycle of 1.92 s. An additional benefit is a reduction in energy consumption [12].

Handharu, et al. [13] studied bipedal pattern generation with toe and heel motion to realize knee stretch walking. The hip trajectory that satisfies ZMP is obtained by the method in [14], and the inverse kinematics is solved given an initial feet trajectories. Then the initial knee trajectories are modified by the cubic interpolation to have the stretched motion. To prevent singularities, the feet motion must be modified accordingly to fit the new knee trajectories. However, in their work, the hip height is constant. The simulated robot has walking speed of 0.63 km/h, with the stride length of 0.7 m and the gait cycle of 4 s.

Morisawa, et al. [15] propose a parametric surface to construct a variable COM motion. The surface is defined as the relative height of the COM to the stance foot position. The variable COM height lowers the hip position at every double support phase, so the robot avoids an over extension of its knees, thus prevents excessive joint speed. This gait is more human-like with a speed of 1.35 km/h, at the stride length of 0.6 m and cycle time of 1.6 s. However, there is no mention of toe-off and heel-strike motion.

Takenaka, et al. [16] achieved a fast walking by using an approximate dynamic model and the technique called "the
divergent component of motion" for the robot Asimo. The three-mass model has a good dynamics approximation than a single mass model, and at the same time avoids detailed modeling. It performs straight walking at the instantaneous maximum speed of 3.08 km/h, at the stride length of 0.96 m and the gait cycle of 1.088 s.

In this paper, we present a fast bipedal walking scheme with large strides which is achieved using a variable hip height, and toe-off and heel-strike motion. To realize this, we relax the constant COM constraint in the cart-table model. Although fast walking might also be achieved by conventional bent knee walking, considering energy efficiency, it is hence desirable to select a stable gait which demands less bending of the knees. This necessitates the design of a hip pattern generator with a variable height trajectory giving higher hip positions in the single support phase to straighten knees, and a lower hip position in the double support phase to prevent the knee singularities. Meanwhile, feet motion with toe-off and heel-strike further enlarges the legs’ workspace. So the hip position doesn’t need to be lowered too much. The singularity is avoidable through the proposed tuning procedure of the hip and feet trajectory generators.

Section II briefly introduces the cart-table model, the preview control method, and the kinematic analysis of the singularity. Section III mainly presents the pattern generators for the hip and feet trajectories respectively. Section IV provides the dynamic simulation of five different gaits with an average speed up to 0.7 m/s (2.52 km/h), which is 140% of full leg length per second (whole leg length is about 0.5 m). We conclude our work in the last section.

II. ANALYSIS OF KINEMATIC CONSTRAINTS

A. Cart-table Model and Preview Control

The cart-table model proposed by Kajita, et al. [7] assumes a simplified robot model where a cart of mass $m$ is placed on a mass-less table. The robot is simplified to a point mass. The ZMP is the point where the resultant inertia force penetrates the ground. If the cart, representing the COM, stays outside of the foot area of the table, the ZMP can still be positioned inside the support region by choosing a proper acceleration, ensuring the ZMP criterion [17]. A preview controller is used to generate the horizontal motion of COM for different footholds.

B. Kinematic Constraints

This section describes why the constant height of the hips is undesirable for large stride walking. This is graphically presented in Fig. 1 (a) which shows that an decrease in COM height avoids singularity in the double support with flat feet. In the conventional approach, the fixed height is reduced such that no singularity occurs at the knee joints, especially at the beginning and the end of the double support. Although this solves the singularity issue, it has another drawback. As illustrated in Fig. 1 (c), when the robot hip passes over the ankle joint in the mid-stance, the knee joint angle becomes large, resulting in high motor torque to counteract gravity.

However, when toe-off and heel-strike motion are considered, the workspace is able to cover the footholds with a higher hip position. More beneficially, knee joint reduces in mid-stance phase, which results in lower torque. Fig. 1 (b) shows how the feet motion prevents the knee singularity when a hip position is higher. Furthermore, Fig. 1 (d) equally demonstrates how raising the hip position in the single support phase minimizes the knee joint angle.

III. DESIGN OF PATTERN GENERATORS

This section introduces mathematical formulation and proves the feasibility of varying the COM height. The spatial COM motion is solved in the horizontal plane by the preview control and in the vertical axis by the hip pattern generator.

A. Robustness to COM Height Variation

A complete ZMP equation including angular momentum is

$$x_{zmp} = \frac{M(z_{com} + g)x_{com} - Mz_{com}z_{com} - \dot{L}_g}{M(z_{com} + g)}$$

$$y_{zmp} = \frac{M(z_{com} + g)y_{com} - My_{com}z_{com} + \dot{L}_x}{M(z_{com} + g)}$$

where $\dot{L}_g$ and $\dot{L}_x$ are the rate of change of angular momentum around COM about $y$ and $x$ axis respectively.

$$L = \sum_{i=1}^{N} c_i \times \vec{p}_i + \sum_{i=1}^{N} (R_i I_i R_i^T) \vec{R}_i \omega_i$$

In (3), the first term is the linear momentum of the $i$th body segment with respect to the overall COM, and the second term is the angular momentum of $i$th body segment around its local COM. $I_i$ and $\omega_i$ are the inertia tensor and angular velocity of the $i$th segment in the corresponding segmental reference frame, and $R_i$ is the transformation matrix of the segment with respect to the global frame. Given the same gait parameters, we assume that the angular momentum terms are nearly the same for a slightly different COM height, and we use the single mass model to compare the error caused by...
the COM height variation. The ZMP equation of a single mass model is

\[ x_{zmp} = x_{com} - \frac{\ddot{x}_{com}}{g} \dot{z}_{com} \]  

(4)

In the state space equation used by the preview controller, the COM height is considered to be constant. The simplified ZMP equation with constant \( z_c \) is

\[ x_{zmp} = x_{com} - \frac{\ddot{x}_{com}}{g} z_{com} \]  

(5)

, where \( z_c \) is a constant in (5) while \( z \) and \( \ddot{z} \) are variables in (4). The ZMP error caused by vertical COM motion is

\[ e_z = x_{zmp} - x_{\text{com}} = \bar{\ddot{x}} z + g(z_c - \bar{z}) / (g(\bar{z} + g)) \]  

(6)

Partial differential equations of ZMP error \( e_z \) are

\[ \frac{\partial(e_z)}{\partial(z)} = \frac{\bar{\ddot{x}}}{\bar{\bar{z}} + g} \]  

(7)

\[ \frac{\partial(e_z)}{\partial(\ddot{z})} = \bar{\ddot{x}} \left( \frac{z_c}{g(\bar{z} + g)} - \frac{z_c + g\bar{z} - g\bar{\ddot{z}}}{g(\bar{z} + g)^2} \right) \]  

(8)

Linearize the partial derivatives around the equilibrium condition \( z_0 \) and \( \ddot{z} = 0 \), the ratio of error caused by \( \Delta \bar{z} \) and \( \Delta \ddot{z} \) are

\[ \left| \frac{\partial(e_z)}{\partial(\bar{z})} \right| \Delta \bar{z} = \frac{\left| \partial(e_z) \right|}{\left| \partial(\bar{z}) \right|} \Delta \bar{z} = \left| \Delta \ddot{z} / g \right| \]  

(9)

From (9), we gain the insight that the acceleration item \( \ddot{z} \) plays a more critical role since the height variation \( \Delta z / z_c \) is relatively small compared to the acceleration variation \( \Delta \ddot{z} / g \). Thus, the ZMP error \( e_z \) introduced by the vertical COM motion is mainly determined by \( \Delta \ddot{z} \).

Conceptually, we treat the height deviation as a parametric disturbance. Provided that the hip trajectory is smooth without jerks and the magnitude of \( \Delta \ddot{z} \) is restricted within a certain bound, our study shows that the preview control scheme is robust to the parameter \( z_0 \). This is because the preview control scheme uses the multi-body ZMP to compare ZMP tracking errors, and compensate these errors in the second preview control loop. Compared to the vertical COM acceleration \( \Delta \ddot{z} \) and the angular momentum effect, the variation of \( \Delta z \) causes negligible ZMP errors.

Additionally, the study of stepping over large obstacles for the HRP-2 robot demonstrates that preview control can be applied even when COM doesn’t stay on a constant height [6]. It suggests that the assumption of the constant height can be relaxed. In that work, the COM height variation is allowed since the second preview loop deals with the disturbances caused by this vertical hip motion.

**B. Hip Pattern Generator**

As shown in Fig. 2, the hip pattern generator describes a wavy hip motion by correlating the hip position in \( x \) and \( z \) axis by

\[ z_{\text{hip}} = z_c + \frac{A}{2} \left[ \cos(2\pi \frac{x_{\text{hip}} - x_{\text{front}}}{x_{\text{front}} - x_{\text{rear}}}) - 1 \right] \]  

(10)

, where \( x_{\text{front}} \) and \( x_{\text{rear}} \) are the ankle position of front and rear foot respectively, \( z_c \) is the initial hip height, and \( A \) is the amplitude of sinusoidal pattern. The higher the \( z_c \) is, the more straightening knees in the single support. The larger \( A \) gives a lower hip position in the double support, therefore it is more likely to avoid the knee singularities. Two parameters \( z_c \) and \( A \) can be tuned to generate various hip patterns. It should be careful to use sinusoidal equation to vary the hip height as a function of time, because this results in a distorted sinusoidal pattern in the Cartesian space, as the horizontal velocity \( \ddot{x}_{\text{hip}} \) is not constant. In contrast, our method defines \( z_{\text{hip}} \) as a function of \( x_{\text{hip}} \) with respect to the foot positions.

Substituting the step length \( L_{\text{step}} = x_{\text{front}} - x_{\text{rear}} \) into (10). The amplitude of acceleration caused by this vertical motion is

\[ a(t) = -A(2\pi/L_{\text{step}})\sin(2\pi \frac{x_{\text{hip}} - x_{\text{front}}}{L_{\text{step}}}) \dot{x}_{\text{hip}}(t) \]  

(11)

If we set a bound of acceleration variation by setting a maximum magnitude \( a_{\text{max}} \), we can formulate an estimation approach to set the parameter \( A \).

\[ |a|_{\text{max}} \leq A(2\pi/L_{\text{step}})\dot{x}_{\text{hip}}(t) \leq A(2\pi/L_{\text{step}})V_{\text{max}} \]  

(12)

We can use the average velocity \( (2L_{\text{step}})/T_{\text{cycle}} \), where \( T_{\text{cycle}} \) is the gait cycle to approximate \( V_{\text{max}} \), yielding

\[ |a|_{\text{max}} \approx A \frac{2\pi}{L_{\text{step}} T_{\text{cycle}}} = A \frac{4\pi}{T_{\text{cycle}}} \]  

(13)

This hip pattern generator utilizes a simple function (10) instead of high order polynomial interpolation, so its derivative is easily obtained in an analytical form as in (11) with the parameter \( A \) bounds the vertical acceleration limit as expressed in (12). Equation (13) briefly gives a close estimation of acceleration boundary given the parameter \( A \) and the gait cycle \( T_{\text{cycle}} \). For instance, a gait cycle of \( T_{\text{cycle}} = 1s \), \( A = 0.02m \) only results in maximum acceleration of \( 0.25m/s^2 \). Also, (13) reveals that \( a_{\text{max}} \) is irrelevant to the stride length, and the parameter \( A \) can also be obtained if we wish to restrict a bound of vertical acceleration \( |a|_{\text{max}} \). In short, the proposed hip pattern generator guarantees a smooth trajectory which has its peak in the mid-stance and its valley in the mid of the double support, and continuous vertical acceleration \( a(t) \) profile without jerks, which can be easily bounded by one parameter \( A \) according to (13).
C. Foot Trajectory Generation

In Fig. 3, when the foot rotates at the maximum angle $\theta_{\text{toe}}$ before it clears the ground, the ankle position is denoted as $(x_{\text{toe}}, z_{\text{toe}})$. When heel strikes ground with the maximum angle $\theta_{\text{heel}}$, the ankle position is denoted as $(x_{\text{heel}}, z_{\text{heel}})$. $(x_0, H_0)$ is the ankle position of the previous stance foot, while $(x_f, H_o)$ is that of the new stance foot. The highest position of the swing foot is $(x_0 + p \cdot L_{\text{stride}}, H)$, where $p$ stands for the percentage $(0 < p \leq 0.5)$.

The method which we apply is similar to the work in [14]. However, we found out that the continues COM acceleration is curial for a stable rapid walking. For fast walking with large strides, any non-zero acceleration at the boundary of transition trajectories would cause considerable acceleration spikes of the COM, thus decreases the stability. Therefore, instead of using cubic spline to connect separate trajectories, we use 5th order polynomial to transfer different walking trajectories smoothly so that both the acceleration and velocity terms are restricted as zero at the boundary condition.

Firstly, the foot rotation is generated by interpolating angle sections $[0, \theta_{\text{toe}}], [\theta_{\text{toe}}, -\theta_{\text{heel}}], [-\theta_{\text{heel}}, 0]$ according to time $t$. Following this, the ankle position $x(t)$ sections such as $[x_0, x_{\text{toe}}], [x_{\text{toe}}, x_{\text{heel}}], [x_{\text{heel}}, x_f]$ are interpolated according to time $t$, where $x_{\text{toe}}, x_{\text{heel}}$ are the ankle positions corresponding to foot rotational angles. Finally, the ankle height $z(t)$ is interpolated according to $[x_0, x_{\text{toe}}], [x_{\text{toe}}, x_0 + p \cdot L_{\text{stride}}], [x_0 + p \cdot L_{\text{stride}}, x_{\text{heel}}], [x_{\text{heel}}, x_f]$. Similarly, the ankle position $y(t)$ can be obtained by interpolating $[y_0, y_f]$ in $[x_0, t_f]$.

D. ZMP Trajectory Generation

Generally, in bipedal walking research, a popular ZMP design in single support is to position the ZMP at the foot center for a maximum safety margin reason [7], [16], [15], [18]. However, this design has larger $\ddot{x}$ variation since the displacement between the COM and the ZMP is larger. In order to enhance stability for the fast walking in large strides, we evolve the ZMP trajectory smoothly from the rear foot to the fore foot, to reduce the $\ddot{x}$ variation.

In Fig. 4, the red dots represent the start and the end point of ZMP transfer in the different phases. In the single support, the ZMP transfers from the back to the front of the stance foot with safety margins, shown in Fig. 4(a). Figs. 4(b), 4(c), and 4(d) are the double support phase consists of heel-strike and toe-off motion. Due to this, the support polygon changes during double support. Nevertheless, safety margins are guaranteed no matter how support polygon varies. The under-actuated foot rotation is avoided for stability reason. So when one foot rotates, the other foot always contacts the ground to form a surface support.

It can be seen that the ZMP and feet trajectories must be designed together, because feet trajectories determine the ground contact. The timing of ZMP transfer is crucial. For example, in Fig. 4(b), from stance to pre-heel-strike, the ZMP shall start to move outside of stance foot only after heel-strike happens. From the post-toe-off to the next stance, the ZMP must move inside new stance foot before the rear foot lifts, as shown in Fig. 4(d).

IV. DYNAMIC SIMULATION OF FAST WALKING

Pattern generators for the feet and the hip trajectories are incorporated in the preview control scheme based on a cart-table model. The overall control scheme and parameter tuning method are similar to the work in [19]. The parameter tuning is simple and intuitive according to the kinematic meanings. Different gaits are generated and validated in the dynamic simulation.

A. Robot Model

We use the proposed scheme for the control of a simulated bipedal robot (Fig. 5(b)), which represents the 3.5 years old child humanoid robot iCub [20], shown in Fig. 5(a). Its upper leg is 0.23m, calf is 0.23m, and ankle height is 0.04m. So iCub only has full leg length of 0.5m, nearly half the size of average adult humans. In OpenHRP3 [21] simulation, the robot model is built according to iCub’s physical parameters.
B. Dynamic Walking

Five different gaits are generated in the successively increased stride lengths. Gait parameters are listed in Table I. Dynamic simulations were implemented in OpenHRP3 to validate the effectiveness. The joint angles and velocities are the references for the joint tracking control.

<table>
<thead>
<tr>
<th>Stride Length</th>
<th>Percent of Full Leg Length</th>
<th>Walking Speed</th>
<th>Normalized by Leg Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3m</td>
<td>60%</td>
<td>0.3m/(1.08km/h)</td>
<td>0.6x/(2.16km/h)</td>
</tr>
<tr>
<td>0.4m</td>
<td>80%</td>
<td>0.4m/(1.44km/h)</td>
<td>0.8x/(2.88km/h)</td>
</tr>
<tr>
<td>0.5m</td>
<td>100%</td>
<td>0.5m/(1.56km/h)</td>
<td>1.0x/(3.12km/h)</td>
</tr>
<tr>
<td>0.6m</td>
<td>120%</td>
<td>0.6m/(2.16km/h)</td>
<td>1.2x/(4.32km/h)</td>
</tr>
<tr>
<td>0.7m</td>
<td>140%</td>
<td>0.7m/(2.32km/h)</td>
<td>1.4x/(5.04km/h)</td>
</tr>
</tbody>
</table>

In the dynamic simulation, the robot iCub successfully performs walking gaits. Fig. 6(a) shows the snapshots of the fast walking gaits in different stride lengths. Walking in large strides with toe-off and heel-strike brings the feature of fast walking, which can hardly be achieved by small stride walking with flat feet.

Furthermore, large gaits enable the robot to step over the wide obstacles, which the small stride walking is unable to perform. Fig. 6(b) shows the robot stepping over a wide obstacle in the stride length of 0.6m, and Fig. 6(c) shows the robot crossing over two wide obstacles in the stride length of 0.7m. The size of the obstacles is considerable wide compared to the size of the child “iCub” robot.

Fig. 7(a) shows quite desired movements in x direction, and the slopes of the traveled distances indicate the walking speeds. Fig. 7(b) reveals the occurrence of slippery rotation along z axis. Given the same friction configuration ($\mu_s=0.8$, $\mu_k=0.5$), the slipping rotation is more likely to happen as the speed goes high. It doesn’t necessarily cause the fall of the robot, since the ZMP criterion has no constraint on the torque $\tau_z$ at the ZMP point. However, it may possibly results in the drifting of the walking direction. Experiments on HRP-2 showed that there was enough friction in real so the slipping in the z direction was noticed only in the simulations but not experiments. Fig. 7(c) shows the sinusoidal patterns of the hip vertical motions. At a higher walking speed, more jerks are observed due to the larger foot ground impacts.

V. CONCLUDING REMARKS

The study suggests that taking large strides is essential to realize the fast bipedal walking. The height-varying hip patterns, and feet trajectories with toe-off and heel-strike enables large strides by relaxing the kinematic constraints. The combination of the preview control and the proposed pattern generators is promising. In addition, large strides not only enhance the fast walking but also allow walking over obstacles.

Although similar motion behaviors such as a sinusoidal hip motion, toe-off and heel-strike, are also observed in biomechanical study of humans [22], we do not claim that the walking manner of this simulated robot is more human-like, since the two systems differ a lot in nature. In contrast, we design the hip and feet trajectories by analyzing the constraints of performing large strides from a kinematics point of view, rather than imitating human motions.

In general, an under-actuated toe-off or heel-strike motion
undoubtedly decreases stability due to a line contact between the foot and ground, and the robot would tip over. Therefore, in the design of the ZMP and feet trajectories, we avoid any under-actuated phase and always position the ZMP inside the polygon with stability margins.

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REFERENCES


